

## UNIT - 3

### Stress Distribution in Soils :-

#### Boussinesq's Theory for point load :-

##### Assumptions:-

- Soil is assumed to be homogenous, isotropic, semi-infinite and elastic.
- Hook's law is valid
- Self weight of soil is neglected
- Soil is initially unstressed
- Any change in the volume of soil due to application of the load is neglected i.e. soil is incompressible.
- The top surface of the soil is free from any shear stress and is subjected to only point load.
- Distribution of the stresses is symmetric along the vertical stress axis.
- Continuity of stress exists in the medium.

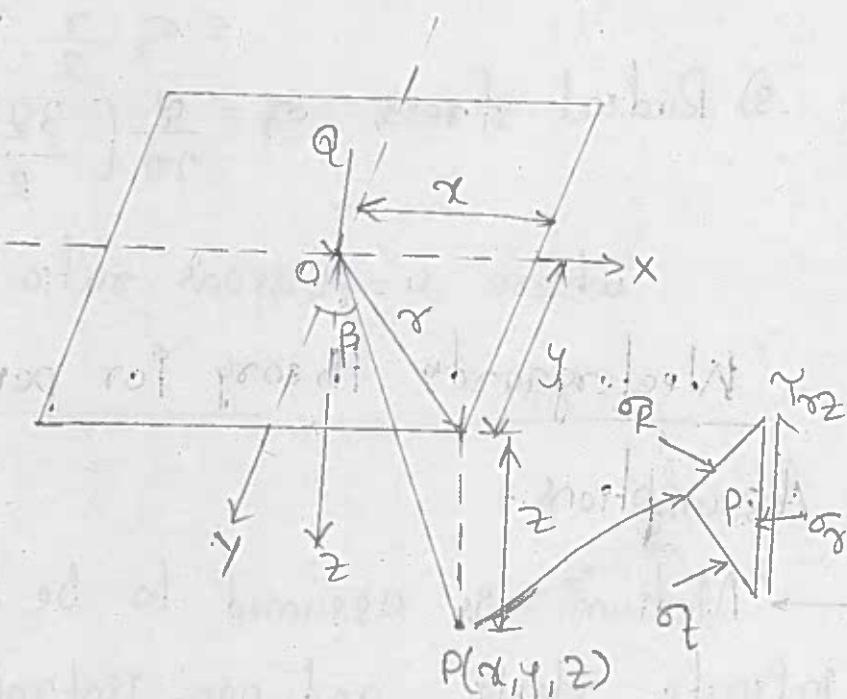
Ref :-

soil Mechanics

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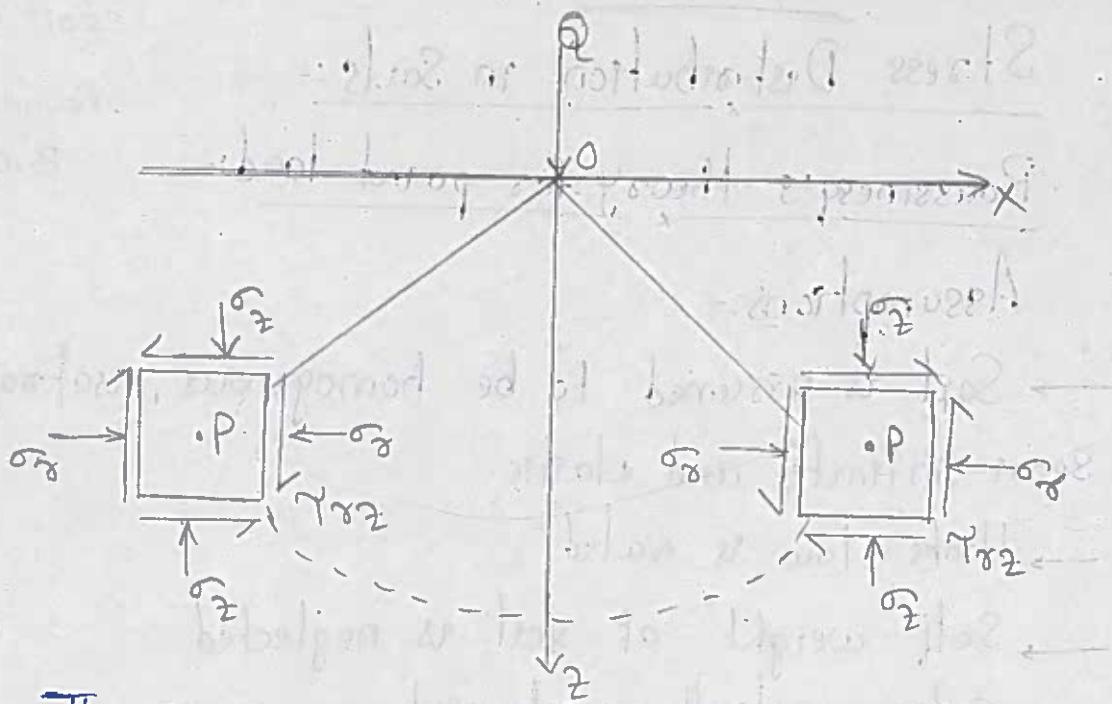
Foundation Engg.

B.C. Punmia.



where,  $\sigma_z = \sqrt{x^2 + y^2}$

$$\sigma_x = \frac{P}{\sqrt{x^2 + y^2}} = \frac{P}{\sqrt{x^2 + y^2 + z^2}}$$



The Boussinesq's equations are as follows -

$$1) \text{ Vertical stress, } \sigma_z = k_B \frac{Q}{z^2}$$

where  $k_B$  = Boussinesq influence factor

$$= \frac{3}{2\pi} \left[ \frac{1}{1 + \left(\frac{r}{z}\right)^2} \right]^{5/2}$$

$k_B$  is a number and is a function of  $r/z$  ratio.

$$2) \text{ Shear stress, } \tau_{rz} = \frac{3Q}{2\pi} \cdot \frac{rz^2}{[r^2 + z^2]^{5/2}}$$

$$= \sigma_z \cdot \frac{r}{z}$$

$$3) \text{ Radial stress, } \sigma_r = \frac{Q}{2\pi} \left[ \frac{3zr^2}{R^5} - \frac{(1-2\mu)}{R(R+z)} \right]$$

where  $\mu$  = Poisson's ratio

### Westergaard's theory for point load :-

Assumptions:-

→ Medium is assumed to be homogenous, semi-infinite, elastic and non-isotropic

→ Here medium is assumed to be laterally reinforced with fibres of negligible thickness i.e,

medium is considered to be rigid horizontally and elastic vertically. It means no lateral deformations but only vertical deformations.

→ Westergaard equation for vertical stress for a point load, for a Poisson's ratio value equal to zero is given by.

$$\sigma_z = k_w \frac{q}{z^2}$$

where,  $k_w$  = Westergaard's influence factor

$$\sigma_z \text{ max} = 0.3183 \frac{q}{z^2}$$

→ It is to be noted that at  $\frac{r}{z} = 1.52$ .

Uniformly loaded circular area:-

For a circular area of radius 'r' loaded uniformly with 'q' / unit area, the vertical stress at a depth 'z' is given as

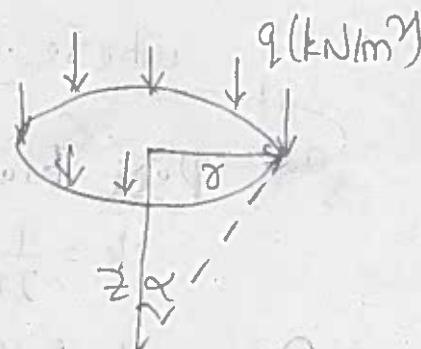
Using Boussinesq's theory :-

$$\text{where } \sigma_z = k \cdot q$$

$$k = 1 - \cos^2 \alpha$$

$$\text{here, } \cos \alpha = \frac{z}{\sqrt{r^2 + z^2}}$$

$$\text{Hence } \sigma_z = \left[ 1 - \left( \frac{1}{1 + \frac{z^2}{r^2}} \right)^{3/2} \right] q$$



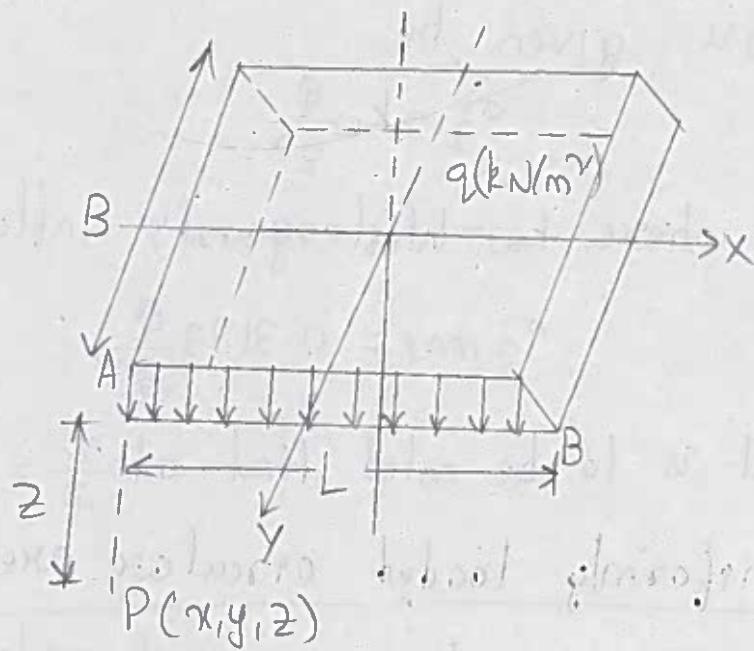
Using Westergaard theory:-

$$\sigma_z = \left[ 1 - \frac{1}{\sqrt{1 + \left( \frac{r}{nz} \right)^n}} \right] q$$

$$\text{where } n = \sqrt{\frac{1+2\mu}{2-2\mu}}, \mu \text{ being Poisson's ratio}$$

## Uniformly loaded Rectangular area:-

\* Vertical stress at a point below the corner of a rectangular area loaded uniformly is given by Newmark's equation.



$$\sigma_z = k \cdot q$$

$$k = \frac{1}{4\pi} \left[ \frac{2mn(m^2 + n^2 + 1)^{1/2}}{m^2 + n^2 + m^2n^2 + 1} \cdot \frac{(m^2 + n^2 + 2)}{(m^2 + n^2 + 1)} \right] + \tan^{-1} \left( \frac{2mn(m^2 + n^2 + 1)^{1/2}}{m^2 + n^2 - m^2n^2 + 1} \right)$$

where,  $m = \frac{L}{2}$  and  $n = \frac{B}{2}$

as per Westergaard theory

$$k = \frac{1}{2\pi} \left( \cot^{-1} \left( \frac{1}{2m^2} + \frac{1}{2n^2} + \frac{1}{4m^2n^2} \right)^{1/2} \right)$$

## Pressure bulb:-

\* Stress isobar is a curve or control contour joining the points of equal vertical pressure in the soil mass.

\* Since the vertical stress on a given horizontal plane is the same in all directions at points

located at equal radial distances from the axis of loading.

- \* The isobar is spatial curved surface and resembles on a onion bulb in shape, it is also called pressure bulb.

- \* With the increasing of depth, vertical stress reduces. Hence generally effect of vertical stress beyond the zone of  $0.2q$  is isobar is neglected.

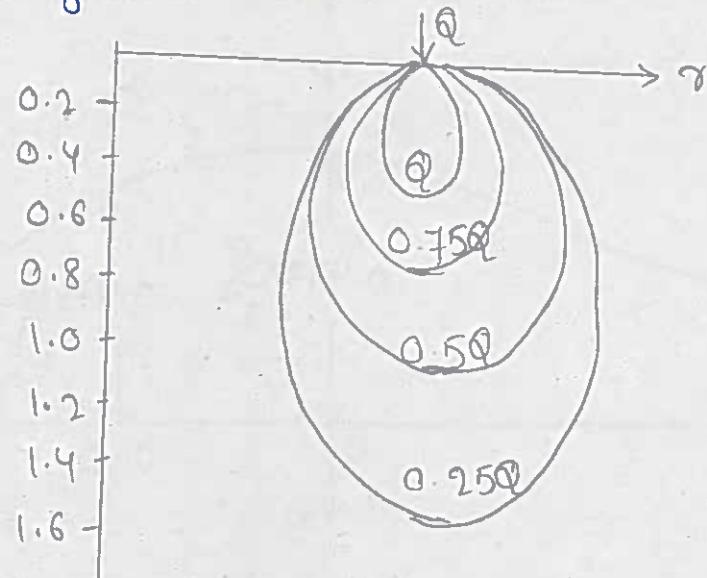


Fig :- Pressure bulb

### Vertical stress distribution on Horizontal plane:-

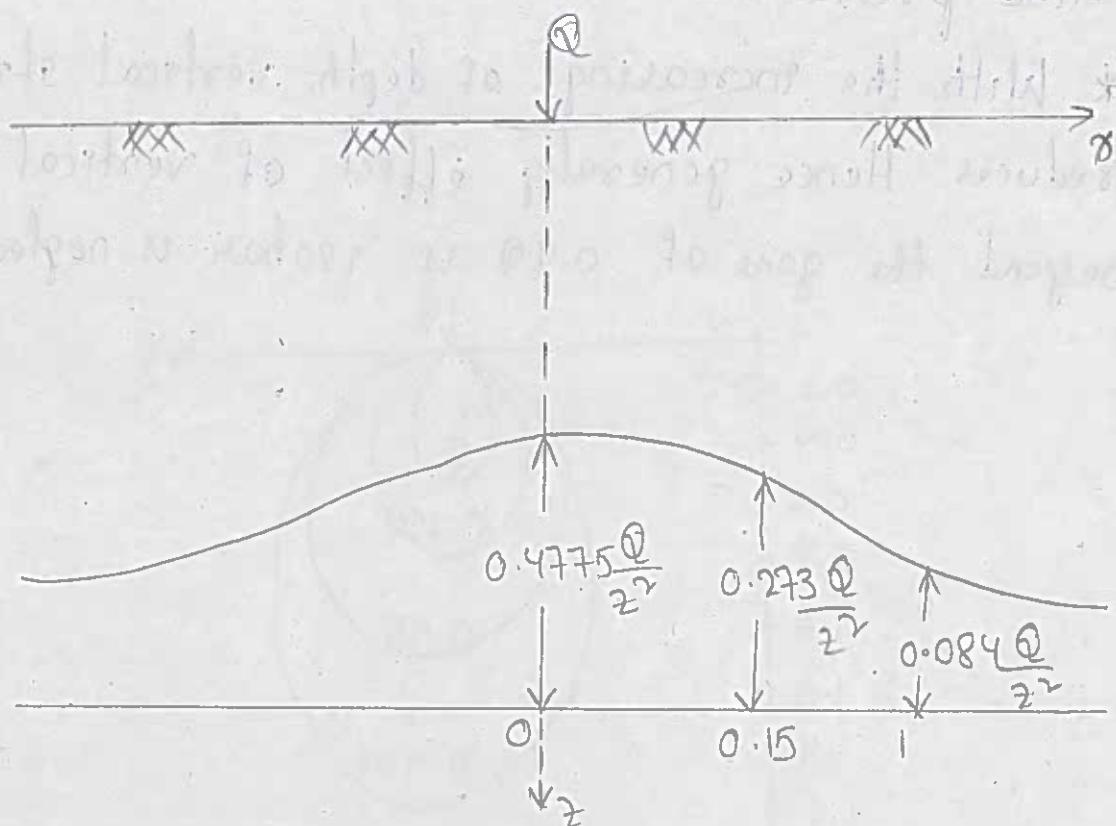
→ Vertical pressure distribution diagram on the horizontal plane at a constant depth of 'z' can be analyzed by using the vertical pressure given by Boussinesq's Theory

$$\sigma_z = k_B \frac{q}{z^2}$$

→  $\sigma_z$  maximum occurs just below the load i.e.,  $z=0$ , which is equals to

$$\sigma_z = 0.477 \frac{q}{z^2}$$

→ Vertical stress distribution diagram on a horizontal plane is bell-shaped; the maximum stress is just below the line of action of the concentrated load and stress decrease asymptotic.



### Vertical stress distribution on Vertical plane:-

→ Vertical pressure on the vertical line first increases with increase in depth and reaches upto its maximum value and start decreasing beyond it with further increase in depth.

Maximum vertical pressure on vertical line occurs when the angle  $B$  made by the polar ray attains a value of  $39^\circ 13' 53.3''$  from the point load.

$$\text{For } \sigma_z \text{ max}, \frac{\partial \sigma_z}{\partial z} = 0$$

$$\frac{\partial}{\partial z} \left[ \frac{3Q}{2\pi} \cdot \frac{z^3}{(z^2 + r^2)^{5/2}} \right] = 0$$

$$6z^2(z^2 + \gamma^2)^{5/2} - 10z^4(z^2 + \gamma^2)^3/2 = 0$$

$$2z^2(z^2 + \gamma^2)^{3/2}(3\gamma^2 - 2z^2) = 0$$

$$3\gamma^2 - 2z^2 = 0$$

$$\frac{\gamma}{z} = \sqrt{\frac{2}{3}} = \tan \beta$$

$$\beta = 39^\circ 13' 53.3''$$

$$\gamma_{xz} = \gamma_2 \cdot \frac{\gamma}{z} = 0.0888 \times \frac{\gamma}{\gamma^2}$$

$$\gamma_{xz} = \frac{\gamma}{\gamma^2}$$

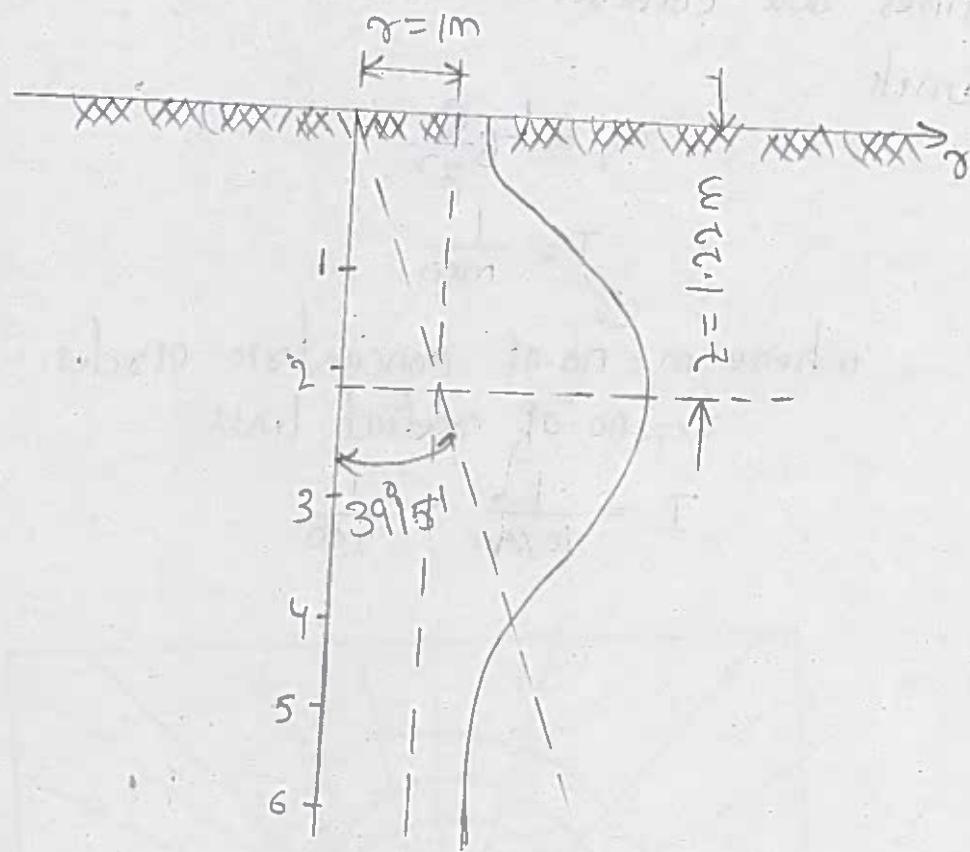


Fig :- Stress on a vertical plane

Newmark's influence chart for irregular areas:-

\* In this method soil is assumed to be homogeneous, semi-infinite, elastic and isotropic

\* These charts are based on Boussinesq's equation so these charts are not applicable for stratified rocks.

\* The greatest advantage of this method is that

it can be used for any shape of the loaded area.

The influence chart consists of a no. of concentric circles and radial lines which divides it into different area units where influence of each area unit at the centre of the chart is same and is referred as "influence factor".

Generally, 10 concentric circles and 20 radial lines are considered which divides into 200 area units.

$$\sigma_z = k_B \frac{Q}{z^2}$$

$$I = \frac{1}{m \times n}$$

where  $m$  = no. of concentric circles.

$n$  = no. of radial lines

$$I = \frac{1}{10 \times 20} = \frac{1}{200}$$

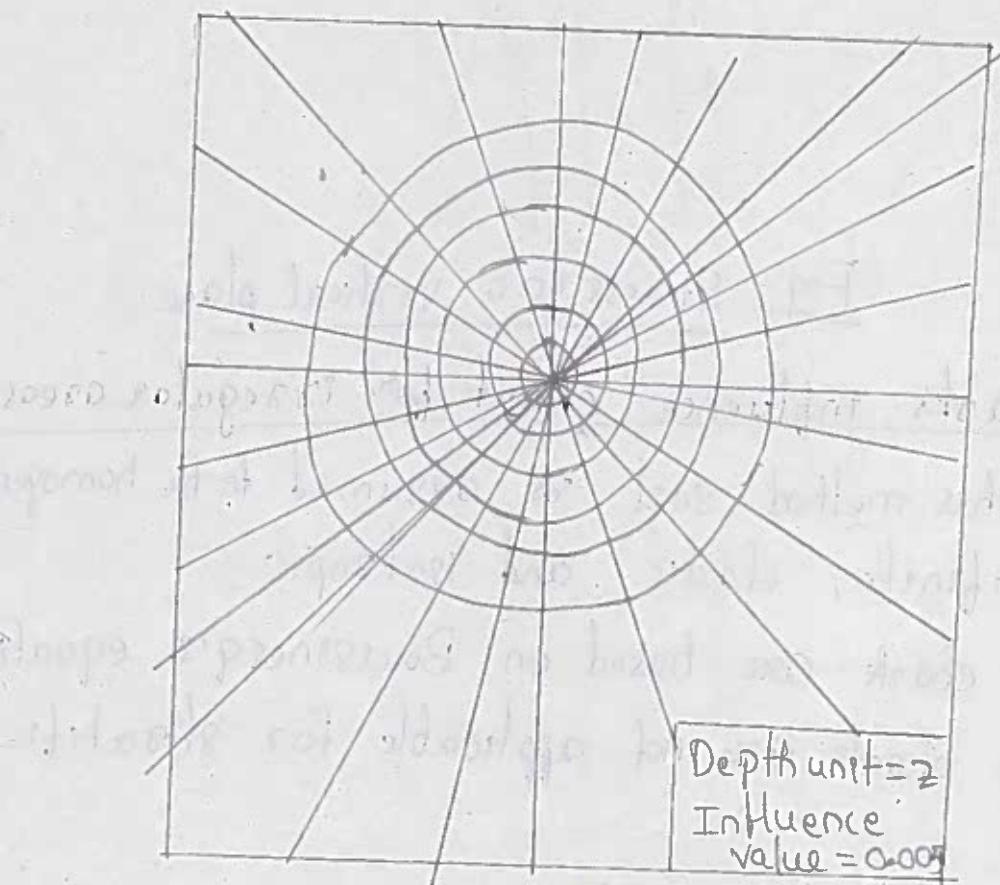


Fig :- Newmark's influence chart

\* In order to use the influence chart for the determination of vertical stresses at any point below a uniformly loaded area of any shape, the following steps are followed.

- 1) Draw the plan of the loaded area with a scale such that the depth  $z$  at which stress is being computed equals the length AB shown on chart
- 2) This plan of the loaded area is then kept on the influence chart such that point at which stress are required, coincides with the centre of chart
- 3) Count the number of area units covered by the plan area on influence chart including the fractional nos.

If  $I_f$  is influence factor.

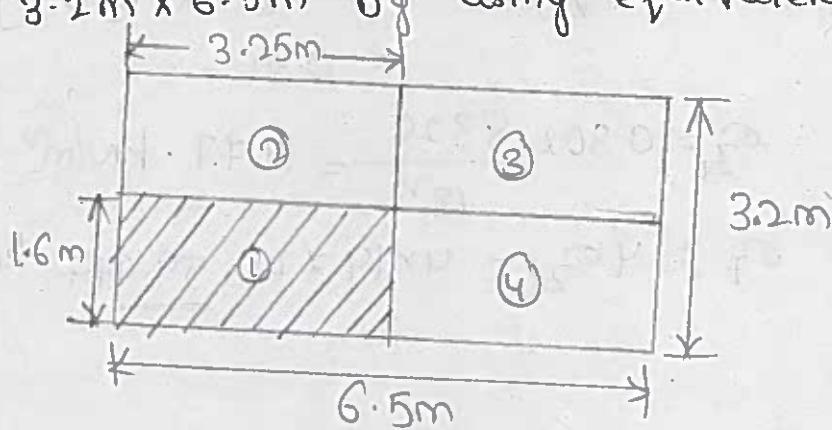
$N_A$  = Total no. of area units occupied

Then vertical stress is given by,

$$\sigma_z = I_f \times N_A \times q$$

$$\sigma_z = \frac{1}{m \times n} \times N_A \times q$$

Q:- Calculate the intensity of stress below the centre of footing at a depth of 13m due to load of 1800 kN/m<sup>2</sup> over the footing area of 3.2m x 6.5m by using equivalent load method.



The radial distance of 'P' from the C.G.  
of divided rectangular areas

$$\gamma_1 = \gamma_2 = \gamma_3 = \gamma_4 = \sqrt{(1.625)^2 + (0.8)^2}$$

$$= 1.81$$

$$z = 13 \text{ m}$$

Equivalent load on the C.G. of Area ①

$$Q_1 = \text{Area} \times q$$

$$= (3.25 \times 16) \times 1600$$

$$= 8320 \text{ kN}$$

Using Boussinesq's equation,

$$\sigma_{21} = k_1 \cdot \frac{Q_1}{z}$$

$$k_1 = \frac{3}{2\pi} \left[ \frac{1}{1 + \left( \frac{\gamma}{z} \right)^2} \right]^{5/2}$$

$$= \frac{3}{2\pi} \left[ \frac{1}{1 + \left( \frac{1.81}{13} \right)^2} \right]^{5/2} = 0.455$$

$$\sigma_{21} = 0.455 \times \frac{8320}{13^2} = 22.4 \text{ kN/m}^2$$

Total stress due to all four smaller triangles

$$\sigma_2 = 4 \times \sigma_{21} = 4 \times 22.4 = 89.6 \text{ kN/m}^2$$

Using Westergaards equation,

$$\sigma_{21} = k_1 \frac{Q_1}{z^2}$$

$$k_1 = \frac{1}{\pi} \left[ \frac{1}{1 + 2 \left( \frac{\gamma}{2z} \right)^2} \right]^{3/2} = \frac{1}{\pi} \left[ \frac{1}{1 + 2 \left( \frac{1.81}{26} \right)^2} \right]^{3/2}$$

$$= 0.30$$

$$\sigma_{21} = 0.30 \times \frac{8320}{13^2} = 14.77 \text{ kN/m}^2$$

$$\sigma_2 = 4 \sigma_{21} = 4 \times 14.77 = \underline{59.077} \text{ kN/m}^2$$

## Compaction

### Mechanism of compaction:-

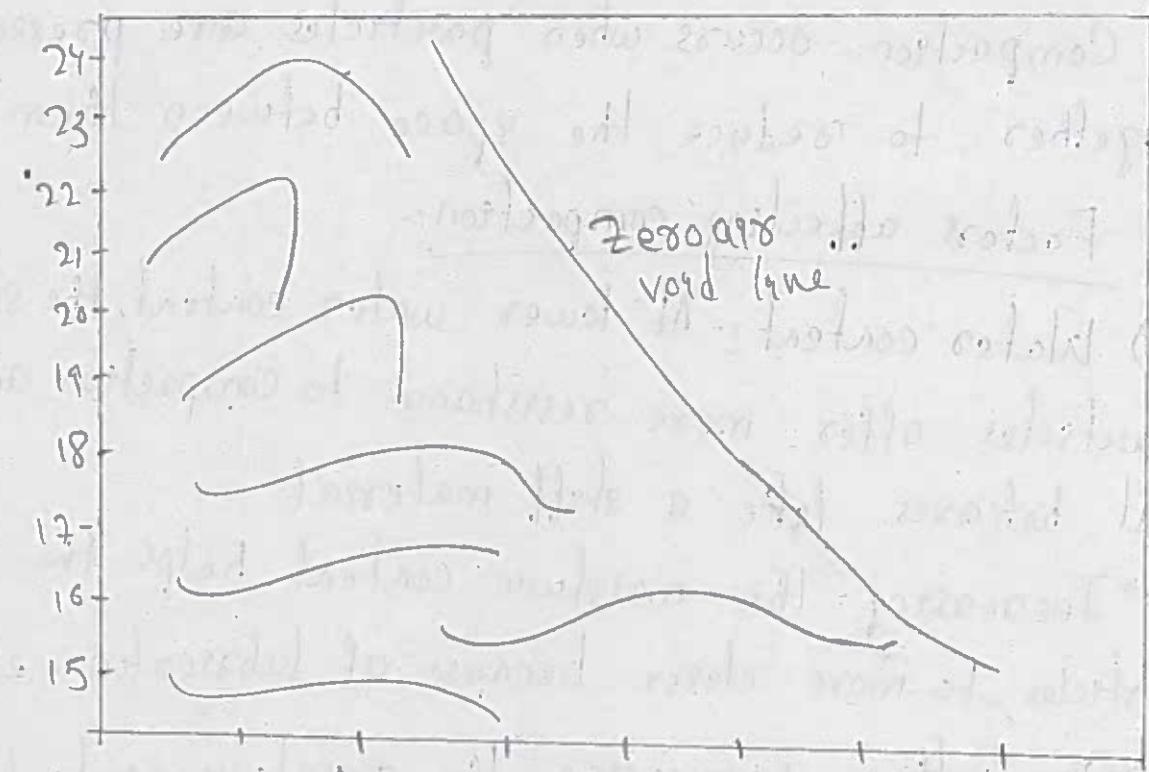
- \* Water is added to lubricate the contact surfaces of soil particles and improve the compressibility of the soil matrix.
- \* It should be noted that increase in water content increases the dry density in most soils up to one stage.
- \* Water act as lubrication.
- \* Compaction occurs when particles are pressed together to reduce the space between them.

### Factors affecting compaction:-

- i) Water content:- At lower water content, the soil particles offer more resistance to compaction and soil behaves like a stiff material.
  - Increasing the moisture content helps the particles to move closer because of lubrication effect
  - On further increasing the moisture content beyond a certain limit, the water starts to replace the soil particles.
- ii) Compactive effort:- For a given type of compaction, an increase in the amount of compaction will initially result in closer packing of the soil particles and maximum dry unit weight increase while the optimum moisture content at which it is attained decreases.

### iii) Type of soil:-

- Coarse grained soils, well graded can be compacted to high dry unit weight especially if they contains some fines.
- However, if quantity of fines is excessive the maximum dry unit weight decreases.
- Poorly graded or uniform sands lead to lowest dry unit weight value.



### iv) Method of compaction:-

- Since the field compaction is essentially a kneading, or rolling type compaction whereas the laboratory tests are dynamic-impact type compaction, therefore, laboratory compaction tests have more value of maximum dry unit weight.

## Effect of compaction on soil properties:-

Soil structure:-

- The water content at which the soils are compacted plays an important role in the engineering properties of soils.
- At lower water content, attractive forces between the particles are stronger than repulsive forces. Hence, soils compacted at a water content less than the optimum water content generally have a flocculated structure.
- Increasing the water content, increases the repulsive forces. Hence, soils compacted at the water content more than optimum water content usually have a dispersed structure.

Permeability:-

- The permeability of a soil depends upon the size of voids. Due to increase in water content for a given compactive effort there is an improved orientation of the particles and corresponding reduction in the size of voids which cause a decrease in permeability.
- If the compactive effort is increased, the permeability of the soil decrease due to increased dry density.

Compressibility:-

- At relatively low stress levels, a soil compacted wet side of optimum is more compressible than the soil compacted dry side of the optimum.
- At high stress levels, the compressibility increase due to breakdown of the structure.

Swelling:-

- A soil on the dry side of optimum has a higher water deficiency and a more random particle arrangement. It can therefore, imbibe more water than a soil on the wet of optimum, therefore more swelling occurs on dry side of the optimum.

Shear strength:-

- The shear strength of compacted soils depends upon the soil type, the moulded water content, drainage conditions, the method of compaction, etc.

Field compaction and equipment:-

- \* In the construction of highway embankments and earth dams, soil is first dumped in the form of loose fills, and then compacted to improve the density and strength characteristics.
- In the field, soil is compacted by applying energy through mechanical equipment. The

energy as is transmitted to the soil by applying pressure in any one of the following:-

- i) Static pressure
- ii) Impact
- iii) Vibration.

Type of equipment	Suitability of soil type	Nature of project
Rammers or Tamper	All soils	In confined areas such as fills and retaining walls
Smooth wheeled rollers	Crushed rocks, sands, gravels	Road construction
Sheep foot rollers	Clayey soils	Core of earth dam
Vibratory rollers	Sands	Embankments for oil storage tanks

### Compaction quality control:-

- \* The compacted dry unit weights attained in the field should be checked occasionally for quality control.
- \* Three methods are generally used to check the in-situ density
  - i) Sand-cone method
  - ii) Rubber-balloon method
  - iii) Nuclear density meters